QUEEN'S UNIVERSITY

DEPARTMENT OF ECONOMICS

MIDTERM EXAMINATION

Economics 390

March 3, 2010 **Instructor:** John M. Hartwick

Time Allowed: 75 minutes.

Do four (4) questions, each of equal value.

1) With a given initial stock of homogeneous oil, "correct extraction" leaves the owner with the most profit. Explain "correct extraction"; Explain "most profit".

2) A competitive, oil-extracting firm experiences a contraction in its current asset size, period by period (L.C. Gray model). Connect this contraction to a regular contraction in the market value of the firm.

3) Samuelson (1961) connects "value contraction" of a competitive extractive firm to "true economic depreciation" and arrives at a "value invariant" tax scheme for a competitive firm. ("Value invariance" is a version of tax neutrality in public finance.) Explain how "true economic depreciation" and "value invariance" work to achieve "neutrality" for the extractive firm.

4) A monopoly oil extractor is indirectly a high-price extractor. Explain. Show that Hotelling's high-price, monopoly oil-extractor is a relatively slower extractor of a given stock of homogeneous oil.

5) The tricky issue in oligopoly behavior over multiple periods is that competitors have no incentive to remain committed to their "best" actions or policies, once arrived at, at the beginning. (They generally can do better to re-neg later on, on what they first agreed they were going to do. Open loop solutions involve naive competitive behavior). Explain.

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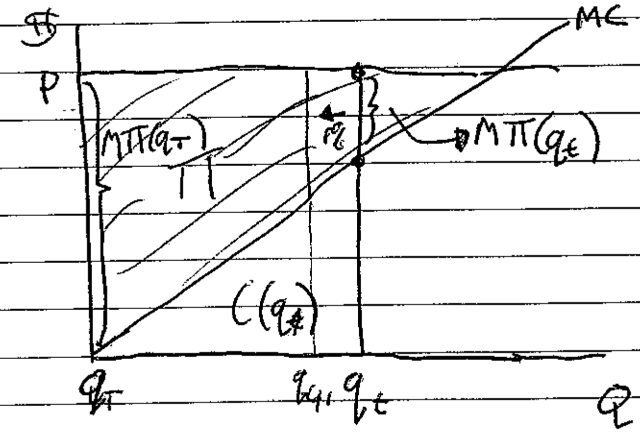
1\. Let us assume a given stock of oil, S_0. The firm extracts q_t in the present period, and thus stock S_{t+1} remains. q_T is the extraction in the last period. Thus: $S_0 = q_t + q_{t+1} \dots q_T$.

To get the "most profit," the owner wants to maximize the present discounted value of all future profits

$$\max \quad \Pi(q_t) + \left(\frac{1}{1+r}\right)\Pi(q_{t+1}) + \left(\frac{1}{1+r}\right)^2 \Pi(q_{t+2}) \dots \left(\frac{1}{1+r}\right)^{T-t} \Pi(q_T)$$

The owner, in a competitive firm, faces a constant price of oil, P, and incurs costs $C(q)$



In each period, the owner thus recieves profits of:

$$\Pi = Pq_t - C(q_t)$$

$$\therefore \frac{d\Pi}{dq_t} = M\Pi = P - MC(q_t)$$

In order to maximize his stream of profits, the firm will equalize the present discounted value of each period's marginal profit.

Therefore, ~~$M\Pi(q_t)$~~ $M\Pi(q_t) = \left(\frac{r}{1+r}\right) M\Pi(q_{t+1})$

or $\frac{d[Pq_t - C(q_t)]}{dq_t} = \frac{1}{(1+r)} \frac{d[Pq_{t+1} - C(q_{t+1})]}{dq_{t+1}}$

This is what is known as the "r% rule": at each period the distance P-MC increases by r% and thus the supplier extracts less oil from the ground in each sucessive period.

This only solves the first part of the problem, however. We know the correct extraction path given any q_t — but how do we know the initial extraction q_t?

To do this, we need to work backwards and maximize $M\Pi$ in the last period. $M\Pi$ is maximized when average profit equals marginal profit. With a linear marginal cost curve, this only occurs when q_T is jammed against the axis (since at any other point, $Av\,\Pi > M\Pi$).

Knowing this q^*_T, we can follow the r% rule all the way back to find the optimal initial extraction, ~~path~~ q^*_t.

In summary, $M\Pi(q^*_t) = \left(\frac{1}{1+r}\right) M\Pi(q^*_{t+1})$ is the "correct extraction ~~expansion~~" that gives the most profit with boundary conditions $Av\,\Pi(q_T) = M\Pi(q_T)$ and $q^*_t + q^*_{t+1} + \ldots q^*_T = S_0$.

2. The asset size of an LC Gray firm at time t is:

$$V(S_t) = \Pi(q^*_t) + \left(\frac{1}{1+r}\right)\Pi(q^*_{t+1}) + \left(\frac{1}{1+r}\right)^2 \Pi(q^*_{t+2}) \ldots\ldots + \frac{1}{1+r}^{T-t}\,\Pi(q^*_T)$$

This is the present discounted value of all future profits, given the optimal extraction path discussed in question 1.

Thus, in period $t+1$, the asset value is:

$$V(S_{t+1}) = \Pi(q^*_{t+1}) + \left(\frac{1}{1+r}\right)\Pi(q^*_{t+2}) + \left(\frac{1}{1+r}\right)^2 \Pi(q^*_{t+3}) \ldots + \left(\frac{1}{1+r}\right)^{T-t+1} \Pi(q^*_T)$$

~~If~~ We multiply $V(S_{t+1})$ by $\frac{1}{1+r}$ and subtract it from $V(S_t)$, we get:

$$V(S_t) - \left(\frac{1}{1+r}\right) V(S_{t+1}) = \Pi(q^*_t)$$

Rearranging,

$$\underbrace{V(S_{t+1}) - \cancel{}\, V_{St}}_{\substack{\text{capital gains} \\ \text{(contraction of asset size)}}} + \underbrace{(1+r)\,\Pi(q^*_t)}_{\text{earnings}} = \underbrace{\overset{\text{market interest rate}}{r}\, V_{St}}_{\text{Market value of firm}}$$

A "smooth-time" version of this same relationship:

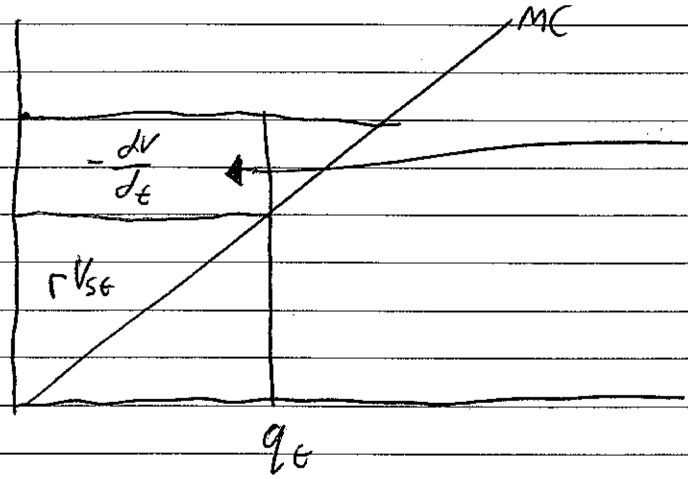
$$\frac{dV_{St}}{dt} + \Pi = r V_{St}$$

~~This dV_{st}~~

Expressed as a rate: $\dfrac{\frac{dV_{st}}{dt}}{V_{st}} + \dfrac{\Pi}{V_{st}} = r$

Thus, capital gains as a rate plus profits as a rate equal the prevailing market rate of interest. Thus, just like a "normal" firm's capital gains + profit equal the rate of return, so too does the extracting firm. Of course, since this is an extracting firm we know that $\frac{dV_{st}}{dt}$ is negative, since the asset shrinks as ~~we~~ the finite stack of oil decreases over time.

Dynamic optimization shows that $-\frac{dV_{st}}{dt} = (P - MC(q_t))\, q_t$



this is the decrease in the firm's value given optimal extraction. This is known as Hotelling "rent" or true economic depreciation

3. A neutral tax causes no inefficiency or distortion because it does not change behaviour. In this case, a Samuelson tax scheme is value invariant because it results in the same optimal extraction path and the value of the taxed firm is equal to the value of the untaxed firm.

There are two major components to the Samuelson tax scheme: (1) true economic depreciation (the shrinkage in the asset value, given optimal extraction) is tax deductible and (2) interest payments are also deductible

Thus, profit for untaxed firm in period t: $\Pi(q^*_t)$
profit for taxed firm: $\Pi(q^*_t) - \tau \Pi(q^*_t) + \tau D_{(t)}$
or $(1-\tau)\Pi(q^*_t) + \tau D_{(t)}$ ↳ economic depreciation

Now lets look at the asset ~~size~~ values of the taxed and untaxed firms:

Untaxed: $V(S_t) = \Pi(q^*_t) + \left(\frac{1}{1+r}\right)\Pi(q^*_{t+1}) + \left(\frac{1}{1+r}\right)^2 \Pi(q^*_{t+2}) \dots \left(\frac{1}{1+r}\right)^{T-t}\Pi(q^*_T)$

taxed: $V^\tau(S_t) = \left[(1-\tau)\Pi(q^*_t) + \tau D_{(t)}\right] + \left(\frac{1}{1+r(1-\tau)}\right)\left[(1-\tau)\Pi(q^*_{t+1}) + \tau D_{(t+1)}\right]$
$\dots + \left(\frac{1}{1+r(1-\tau)}\right)^{T-t}\left[(1-\tau)\Pi(q^*_T) + \tau D_{(T)}\right]$

The intuition that these two schemes are equal comes from the fact that although the profits in each period are smaller, this is "cancelled out" by the less heavy discount term. This less heavy disount term, $\frac{1}{1+r(1-\tau)}$, is a result of interest payment deductability.

This can be seen more clearly with some calculus:

$$V^\tau(S_t) = \int_t^T \left[(1-\tau)\Pi(q^*_{(z)}) + \tau D_{(z)}\right] e^{-r(1-\tau)(z-t)} \, dz$$

Taking the derivative:

$$\frac{dV^\tau(S_t)}{dt} \; (1-\tau) V(S_t) - \left[(1-\tau)\Pi(q^*_t) + \tau D_{(t)}\right]$$

Now, the Key step is ~~using Samuelson's~~ understanding that the true economic depreciation is the shrinkage in asset size, $\frac{-dV^{\tau}(S_t)}{t}$. So, we plug that term in for D_t:

$$\frac{dV^{\tau}}{dt} = (1-\tau) rV(S_t) - \left[(1-\tau)\pi(q^*_t) + \tau\left(-\frac{dV^{\tau}(S_t)}{t}\right)\right]$$

Rearranging:

$$(1-\tau)\frac{dV^{\tau}}{dt} = (1-\tau) rV(S_t) - (1-\tau)\pi(q^*_t)$$

Dividing through by $(1-\tau)$

$$\frac{dV^{\tau}}{dt} = rV(S_t) - \pi(q^*_t)$$ $\Rightarrow$ this is the exact same expression for the shrinkage in the asset in the no tax case shown in question 2.

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Thus, the value of S_t and the optimal path is left unchanged by a tax scheme that makes true economic depreciation and interest payments tax deductible.

Thus, the Samuelson tax scheme is a neutral tax.

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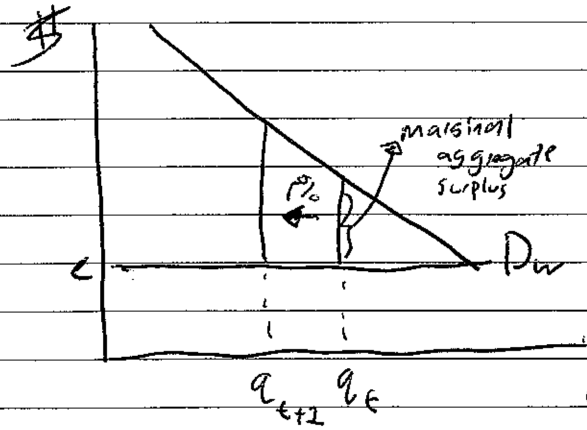
4. The world demand for oil is D_w and there is a constant unit cost of extraction, c



B = benefit, total area under demand curve

To see the contrast in extraction paths between the monopolist and the competitive industry, we can first show the optimal extraction path of the competitive industry.

Welfare, $W = B(Q_t) - cQ_t + \left(\frac{1}{1+r}\right)[BQ_{t+1} - CQ_{t+1}] + \ldots \left(\frac{1}{1+r}\right)^{T-t}[B(Q_T) - cQ_T)$

We also know that ~~competitive~~ $\underbrace{q_t}_{\text{extraction}} = \underbrace{S_t}_{\text{stock today}} - \underbrace{S_{t+1}}_{\text{stock tomorrow}}$

So, $W = B(S_t - S_{t+1}) - C(S_t - S_{t+1}) + \left(\frac{1}{1+r}\right)\left[B(S_{t+1} - S_{t+2}) - C(S_{t+1} - S_{t+2})\right] +$

$$\left(\frac{1}{1+r}\right)^2 \left[B(S_{(t+2)} - S_{t+3}) - C(S_{t+2} - S_{t+3})\right] \ldots$$

Competitive industry maximizes welfare so lets set $\frac{\partial W}{\partial S_{t+2}} = 0$

$$\frac{\partial W}{\partial S_{t+2}} = 0 + \left(\frac{1}{1+r}\right)\left[\underset{q_{t+1}}{B}(-1) - C(-1)\right] + \left(\frac{1}{1+r}\right)^2\left[\underset{q_{t+2}}{B}(1) - C(1)\right] + 0 = 0$$

$$\therefore B(q_{t+2}) - C = \left(\frac{1}{1+r}\right)\left[B(q_{t+2}) - C\right]$$

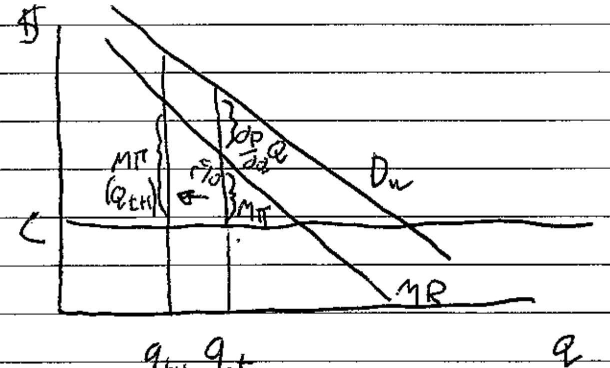
Thus, in a competitive industry, marginal aggregate surplus moves ~~shifts~~ by $r\%$. This optimal extraction path is shown on the graph on the previous page

for a monopolist, $\Pi = P(Q)Q - CQ$

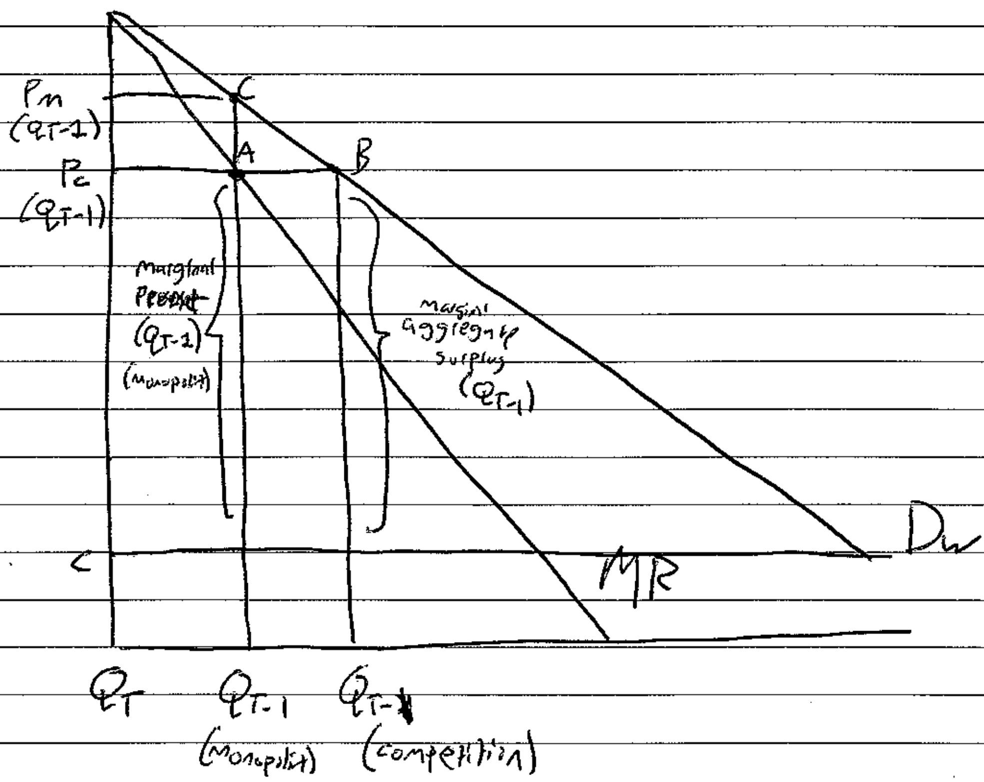
$$\therefore M\Pi = P(Q) + \frac{dP}{dQ}Q - C$$

$$\therefore M\Pi = MR - MC$$

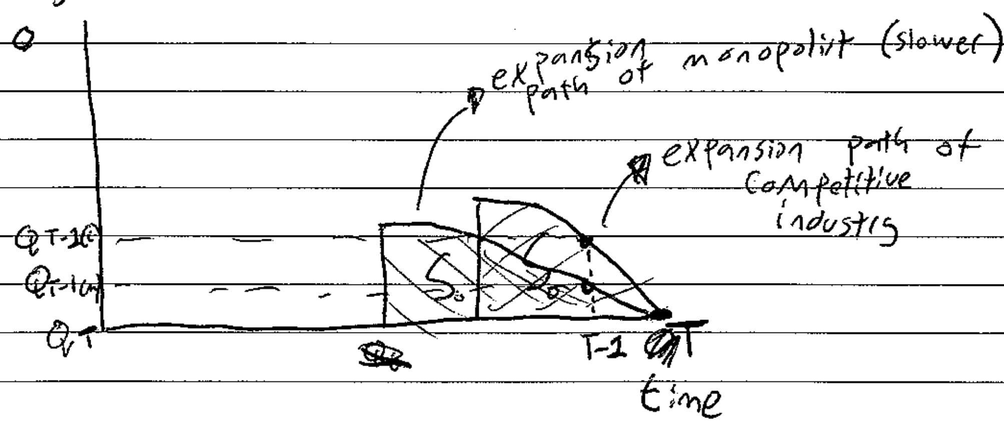
Thus, an optimal expansion path for a monopolist involves moving $MR - MC$ by $r\%$ in each period



We can now compare the two extraction paths. To satisfy the boundary condition that $M\Pi = AV\Pi$, both the competitive industry and the monopolist ~~start~~ end with Q_T jammed against the axis.



We can thus see graphically that shrinking aggregate surplus by r% results in a larger Q_{T-1} than does shrinking the monopolists $M\Pi = MR - C$. Thus, in the second last period, the monopolist extracts less oil but charges a higher price than the competitive industry. ~~The~~ Ironically, then, we see the ~~root of~~ ~~it~~ truth in the Hotelling aphorism "the monopolist is the friend of the conversationist" since in each period they extract less oil.



~~Stiglitz points out that if the demand curves has form~~

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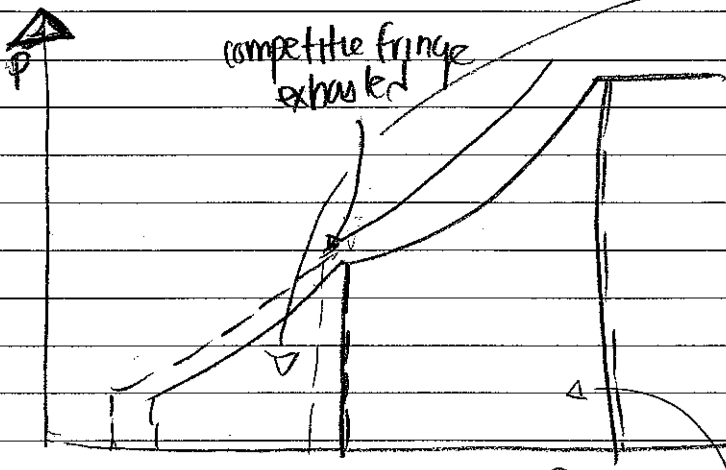
4. If we look at a monopoly like OPEC and a competitive fringe in the oil industry, they will both commit to the plan of an extraction policy in the first phase. The monopoly commits to this plan initially because it wants to see the present value of it's profits. By 'faking' as a competitive firm or in other words dedicating some of its stock to be extracted in the first phase, it ~~will eventually exhaust the supply~~ of the competitive fringe can speed exhaustion

expand detail

$$\frac{\frac{d[P_t - c]}{dt}}{P_t - c} = r$$



phase 1 — monopoly + competitive fringe

phase 2 — monopoly alone

However, once the competitive fringe has been exhausted, there is little incentive for the monopoly to continue on with this extraction policy as it is not profit maximizing. It will then switch to

$$\frac{\frac{d(MR - c)}{dt}}{MR - c} = r$$

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By maximizing period to period, the monopoly has taken advantage of the given situation rather than staying committed to the stated agreement at the beginning. This dynamic inconsistency is the approach often used by oil extracting firms nowadays.